NEAR-NASH TARGETING STRATEGIES FOR HETEROGENEOUS TEAMS OF AUTONOMOUS COMBAT VEHICLES

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ABSTRACT

The Nash equilibrium concept of non-zero sum games is one possible option available to military planners seeking strategies to control large numbers of autonomous assets operating in an adversarial environment. To implement the Nash strategies inherently necessitates making assumptions on possible adversarial actions. However, the Nash concept suffers from one major difficulty which limits its potential usefulness. A Nash equilibrium may not always exist in pure strategies. In this paper we introduce the concept of Near-Nash strategies as a mechanism to overcome this difficulty. We then illustrate this concept by deriving the Near-Nash strategies for a military game where a unique Nash is not guaranteed to exist. We use these strategies as the basis for an intelligent battle plan for heterogeneous teams of autonomous combat air vehicles in the Multi-Team Dynamic Weapon Target Assignment model.

Keywords: Non-zero Sum Games, Nash Strategies, Near-Nash Strategies, Autonomous Combat Vehicles, Target Assignment Problem.

1. INTRODUCTION

As autonomous systems mature from theoretical capabilities into combat ready reality military strategists have become increasingly interested in finding efficient command and control solutions which are capable of intelligently aiding battlefield commanders responsible for large numbers of autonomous assets (Gerkey and Mararik, 2004; Diaz et. al., 2003, 2006; Steinberg, 2006; Kumar et. al., 2006; Chandler, 2004). Lacking the vital improvisational abilities of their human counterparts, these autonomous assets require more in depth battle plans and access to robust mission re-planning. Even as the military migrates from conventional forces into smaller, modular, and consequently more manageable teams of assets, the additional planning needs of an autonomous asset can greatly encumber a commander. Unchecked, this increased workload could overwhelm the capabilities of a battlefield commander, greatly reducing the effectiveness of the asset. Automated battle command aids which use game theoretic strategies are one option which shows considerable promise (Galati and Simaan, 2007; Cruz et. al., 2001, 2004; Ganapathy and Passino, 2003; Liu et. al. 2003a,b). Because possible adversarial actions are inherently considered in a game theoretic analysis, these planners are able to adapt and react to potential enemy actions in an ever-changing battle space. Nash strategies (Nash 1950; Basar and Olsder, 1995) which represent an equilibrium in which neither side benefits from unilaterally deviating from a given strategy pair, provide the predominant methodology for these efforts. Despite their potential usefulness, planners which rely on the Nash equilibrium suffer from one major difficulty. A Nash equilibrium in pure strategies is not always guaranteed to exist. As a result, it is important that the planner be designed in such a way as to be able to handle such a situation. The search for Nash strategies can be very time consuming especially in games where the decision spaces are very large. These planners must therefore have an alternative search strategy that takes care of the possibility of nonexistence of Nash Strategies.

In this paper we introduce the concept of Near-Nash strategies as a mechanism to overcome this difficulty. We then illustrate this concept by deriving the Near-Nash strategies for heterogeneous teams of autonomous combat air vehicles as an attempt to intelligently aid a commander in the planning of a military air operation. We explore the Multi-Team Dynamic Weapon Target Assignment model, and the Near-Nash strategy concept to compute an intelligent battle plan which accounts for the possible actions of the enemy.

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2. NASH STRATEGIES IN BATTLEFIELD SCENARIOS

A military battlefield is an extremely demanding environment. The combination of uncertainty, deceit, and the fluidity of a scenario can cause great difficulty even for the most seasoned military commander. History has shown that the strategies that commanders employ often have a larger impact on the outcome of a battle than the composition and sizes of the forces at their disposal. Often, a brilliant strategy has seized victory from seemingly impossible odds. On the other hand, errors in judgment have also resulted in unexpected disaster.

History has also shown that scenarios in which a smaller force defeated a larger force are often the result of asymmetrical situational awareness (Smith 2007). In many cases, the winning commander was found to have an accurate picture of the force layout and strategy of his/her adversary while the losing commander either did not account for, made many incorrect assumptions, about the adversary. As a result, the losing commander's actions were often not effective or even counter productive while the winning commander's strategies worked to great effect; thus allowing the smaller force to overcome the larger force.

Autonomous assets are in a sense naïve entities, lacking the vital improvisational skills inherent to their human counterparts. These assets are expected to operate in a chaotic, hostile, and ever changing battlefield; the same battlefield that has proven to be so difficult to their human counterparts. Because of this inherent naïveté, it is likely that any human commander will have a much more accurate and complete view of the battlefield than an autonomous asset. As a result, unmanned assets are far more vulnerable to misdirection and are more likely to be deceived. There is a significant danger of an intelligent adversary confusing an autonomous asset in such a way as to induce it to perform in an ineffective or even counterproductive manner.

To achieve maximum effectiveness, automated planners must find robust strategies that take this inherent naïveté into account. While it is highly unlikely that autonomous planners will outperform their human counterparts in the near future, these planners must attempt to mitigate the risk of an autonomous asset being induced to act in a counterproductive manner.

For this reason, the Nash equilibrium (Nash 1950) of non-zero sum games has been a natural approach when automating the decision making process in automated battle field planners (Galati and Simaan, 2007; Cruz et. al., 2001, 2004; Ganapathy and Passino, 2003; Liu et. al. 2003). It is defined for scenarios involving multiple decision makers, each having their own decision space and objective function which generates a measure of the attractiveness of each possible combination of decisions for each decision maker. Furthermore, each decision maker is assumed to have either absolute knowledge or an estimate of the objective function of the other decision maker. Under these conditions a pair of strategies for two decision makers engaged in a game is a Nash equilibrium if no decision maker has an incentive to unilaterally alter its given strategy. In other words, each decision makers' strategy is optimal for the assumed strategies of the other decision maker.

Nash strategies have an inherent robustness which makes them attractive to autonomous planners. A Nash strategy is not optimal in the global sense; a property which often precludes it from achieving the best possible outcome. It is optimal in the sense that it represents the best possible strategy against an intelligent opponent who is acting in a similar manner. While it is true that there is no guarantee that an adversary will act in an intelligent manner, it is possible to say (with a sufficiently accurate scenario model), that an adversary who acts in a manner other than Nash will produce for itself a worse outcome than if it had acted in accordance with a Nash strategy.

3. DIFFICULTIES WITH THE NASH STRATEGIES

The Nash equilibrium has many attractive properties which would seem to promote its widespread implementation. However, the traditional implementation of the Nash equilibrium has two significant drawbacks which have limited its use in many practical scenarios: (1) potential non-existence and (2) potential non-uniqueness. As an illustrate example consider the following non-zero sum game between Players A and B employing strategies $u_A \in \{1,2,3,4\}$ and $u_B \in \{1,2,3,4\}$ as shown in Table 1. Player A wants to maximize $J_A(u_A, u_B)$ and Player B wants to maximize $J_B(u_A, u_B)$. In Table 1 the * denotes the optimal choice for a player given the particular choice of strategy by the other player. For example, if Player B chooses $u_B = 1$, then Player A's optimal choice is $u_B = 1$ as denoted by the * on $J_A^*(1,1) = 5$. Clearly, we see that the strategy pair $\{u_A, u_B\} = \{2,2\}$ is a Nash equilibrium as neither player may benefit from unilaterally deviating from its strategy in this pair.

We should note that while the construction of this matrix is somewhat simple for many applications, the relatively unstructured nature of military conflict combined with the possibly large number of available assets creates a vast and complex decision space, even for relatively small scenarios. Determining the Nash equilibrium in such a space within the time frame of the evolution of the battlefield becomes a significantly challenging problem (Galati, Simaan and Liu, 2003).

A potential issue associated with the Nash equilibrium in planning applications for autonomous combat vehicles is that there is no general guarantee that a single Nash Equilibrium in pure strategies always exists for a given scenario. Examining the matrix shown in Table 1, we can easily construct examples in which the optimal reaction sets do not intersect as well as examples in which they intersect at more than one point.

Table 1 – Sample Game Matrix with a Single Nash

	$u_B = 1$	$u_B = 2$	$u_B = 3$	$u_B = 4$
$u_A = 1$	$J_A^*(1,1) = 5$ $J_B(1,1) = 15$	$J_A(1,2) = 6$ $J_B(1,2) = 4$	$J_A(1,3) = 14$ $J_B(1,3) = 1$	$J_A(1,4) = 0$ $J_B^*(1,4) = 20$
$u_A = 2$	$J_A(2,1) = 1$ $J_B(2,1) = 7$	$J_A^*(2,2) = 10$ $J_B^*(2,2) = 8$	$J_A(2,3) = 4$ $J_B(2,3) = 4$	$J_A(2,4) = 6$ $J_B(2,4) = 2$
$u_A = 3$	$J_A(3,1) = 4$ $J_B(3,1) = 6$	$J_A(3,2) = 2$ $J_B^*(3,2) = 7$	$J_A(3,3) = 15$ $J_B(3,3) = 5$	$J_A^*(3,4) = 8$ $J_B(3,4) = 3$
$u_A = 4$	$J_A(4,1) = 3$ $J_B^*(4,1) = 10$	$J_A(4,2) = 9$ $J_B(4,2) = 2$	$J_A^*(4,3) = 16$ $J_B(4,3) = 5$	$J_A(4,4) = 7$ $J_B(4,4) = 5$

For example, if the value of $J_B(2,4)$ in Table 1 is altered from 2 to 10, the game will no longer have a Nash equilibrium. On the other hand if $J_B(3,4)$ is altered from 3 to 8, the game will have two Nash equilibria. This presents a problem: a Nash strategy is only robust because it is the optimal response to a given strategy. This robustness property disappears if there is no incentive for ones adversary to select that strategy, or if there is an incentive to select another. Game theory makes no prediction as to the outcome if two players choose strategies from different Nash equilibrium points.

In non-zero sum matrix games with random entries it has been shown that the probability of existence of exactly κ Nash equilibria is $p_{\text{Nash}}(\kappa) = e^{-1}/\kappa!$ as the size of the game becomes infinitely large (Stanford 1995). Thus in random games, the probability of existence of more than one Nash equilibrium is 26% and the probability of the game having no Nash equilibrium is 37%. While we recognize that military engagements can hardly be modeled as random games, we can infer that there is a considerable risk that both non-unique and non-existent Nash equilibria may occur even in situations where only the adversary's entries in the matrix are random (Peterson and Simaan, 2008).

4. NEAR-NASH STRATEGIES

When a Nash equilibrium does not exist, one is tempted to look for alternative strategies that may have similar properties. This is problematic because game theoretic models do not share the same principles as standard optimization problems. Unlike classical optimization problems in which the objective function are generally quadratic (concave) around the optimum, implying that strategies located near the optimal point can be assumed to be near optimal, the Nash strategies are defined only as an equilibrium point, and do not necessarily possess a concavity property for nearby strategies. This means that while it is often acceptable to use strategies that are near the optimal strategy in classical optimization problems, it is difficult to predict the outcome of strategies that are near a Nash equilibrium.

There has been some work in dealing with games with no Nash equilibrium. The most significant advancement has been the Epsilon-equilibrium (Everett 1957). Most commonly applied to stochastic games, an Epsilon-equilibrium is defined by a constant ε. A strategy pair is said to be an Epsilon-equilibrium for a given ε if no player can improve its objective function by more than ε by unilaterally deviating from the given strategy. While this is a useful strategy concept, it is not possible to guarantee a priori that an Epsilon-equilibrium exists for a given ε. Selecting an ε that is too high may result in a less intelligent strategy choice than is otherwise available. On the other hand, selecting an ε that is too low may result in cases where no satisfying strategies may be found.

To alleviate this problem, we propose a new concept which we refer to as the Near-Nash equilibrium which expands upon the Epsilon-equilibrium similar to the way which optimization problems benefit from the concept "near-optimal" solutions. Essentially, we reformulate the Nash criteria as an optimization problem which seeks to minimize the squared sum of the losses that each decision maker may obtain by unilaterally deviating from a given pair of strategies.

To mathematically define this concept of a Near-Nash equilibrium consider a two player game between two decision makers, Players A and B. Assume that Player A's optimal response to given strategy u_B of Player B is $u_A^*(u_B)$, which is determined as follows:

$$J_A\left(u_A^*\left(u_B\right), u_B\right) = \max_{u_A \in U_A} J_A\left(u_A, u_B\right) \tag{1a}$$

Similarly, Player B's optimal response $u_B^*(u_A)$ to a particular strategy u_A of Player A, can be derived from:

$$J_B\left(u_A, u_B^*\left(u_A\right)\right) = \max_{u_B \in U_B} J_B\left(u_A, u_B\right) \tag{1b}$$

Thus, the amount player A can lose by unilaterally altering its strategy from the optimal response to a given strategy u_B of player B is:

$$\Delta_A(u_A, u_B) = J_A(u_A^*(u_B), u_B) - J_A(u_A, u_B) \tag{2a}$$

Similarly, Player B's loss by unilaterally altering its strategy from the optimal response to a given strategy u_A of player A is:

$$\Delta_B(u_A, u_B) = J_B(u_A, u_B^*(u_A)) - J_B(u_A, u_B) \tag{2b}$$

We note that $\Delta_A(u_A,u_B)$ and $\Delta_B(u_A,u_B)$ are clearly non-negative quantities. We also note that a Nash equilibrium pair $\{u_A^N,u_B^N\}$ (whether unique or not) may be necessarily and sufficiently defined by the conditions $\Delta_A(u_A^N,u_B^N)=\Delta_B(u_A^N,u_B^N)$ =0; or

$$J_A(u_A^*(u_B^N),u_B^N)-J_A(u_A^N,u_B^N)=0 \quad (3a)$$

$$J_B(u_A^N,u_B^*(u_A^N))-J_B(u_A^N,u_B^N)=0 \quad (3b)$$

As an illustration, for the game defined in Table 1, the value of $\Delta_A(u_A,u_B)$ and $\Delta_B(u_A,u_B)$ are shown in Table 2:

Table 2: Values of $\Delta_A(u_A,u_B)$ and $\Delta_B(u_A,u_B)$ for the game of Table 1

	$u_B=1$	$u_B=2$	$u_B=3$	$u_B=4$
$u_A=1$	$\Delta_A(1,1)=0$ $\Delta_B(1,1)=5$	$\Delta_A(1,2)=4$ $\Delta_B(1,2)=16$	$\Delta_A(1,3)=2$ $\Delta_B(1,3)=19$	$\Delta_A(1,4)=8$ $\Delta_B(1,4)=0$
$u_A=2$	$\Delta_A(2,1)=4$ $\Delta_B(2,1)=1$	$\Delta_A(2,2)=0$ $\Delta_B(2,2)=0$	$\Delta_A(2,3)=12$ $\Delta_B(2,3)=4$	$\Delta_A(2,4)=2$ $\Delta_B(2,4)=6$
$u_A=3$	$\Delta_A(3,1)=1$ $\Delta_B(3,1)=1$	$\Delta_A(3,2)=8$ $\Delta_B(3,2)=0$	$\Delta_A(3,3)=1$ $\Delta_B(3,3)=2$	$\Delta_A(3,4)=0$ $\Delta_B(3,4)=4$
$u_A=4$	$\Delta_A(4,1)=2$ $\Delta_B(4,1)=0$	$\Delta_A(4,2)=1$ $\Delta_B(4,2)=8$	$\Delta_A(4,3)=0$ $\Delta_B(4,3)=5$	$\Delta_A(4,4)=1$ $\Delta_B(4,4)=5$

Similarly, if we compute these values for the modified game where the value of $J_B(2,4)$ in Table 1 is altered from 2 to 10, the corresponding table will be identical to Table 2, except for the second row which will change as illustrated in Table 3. We note that this modifies game has no Nash equilibrium.

Clearly, the fact that this Table has no {0,0} entry confirms that the game has no Nash solution. Alternatively, assume that the two players wish to find a pair of strategies which guarantees each player minimum losses is the other player deviates from its optimal reaction to its strategy. Since the losses consist of a pair of numbers, we define a measure of the cumulative loss by both players by the expression:

$$J(u_A,u_B)=\Delta_A^2(u_A,u_B)+\Delta_B^2(u_A,u_B) \quad (4a)$$

or

$$J(u_A,u_B)=\left[J_A(u_A^*(u_B),u_B)-J_A(u_A,u_B)\right]^2 + \left[J_B(u_A,u_B^*(u_A))-J_B(u_A,u_B)\right]^2 \quad (4b)$$

This suggests that just as a Nash equilibrium is characterized by a pair of strategies in which neither player can gain by unilaterally deviating from it, a strategy pair which minimizes both players' cumulative

Table 3: Values of $\Delta_A(u_A,u_B)$ and $\Delta_B(u_A,u_B)$ for the modified game of Table 1

	$u_B=1$	$u_B=2$	$u_B=3$	$u_B=4$
$u_A=1$	$\Delta_A(1,1)=0$ $\Delta_B(1,1)=5$	$\Delta_A(1,2)=4$ $\Delta_B(1,2)=16$	$\Delta_A(1,3)=2$ $\Delta_B(1,3)=19$	$\Delta_A(1,4)=8$ $\Delta_B(1,4)=0$
$u_A=2$	$\Delta_A(2,1)=4$ $\Delta_B(2,1)=3$	$\Delta_A(2,2)=0$ $\Delta_B(2,2)=4$	$\Delta_A(2,3)=12$ $\Delta_B(2,3)=6$	$\Delta_A(2,4)=2$ $\Delta_B(2,4)=0$
$u_A=3$	$\Delta_A(3,1)=1$ $\Delta_B(3,1)=1$	$\Delta_A(3,2)=8$ $\Delta_B(3,2)=0$	$\Delta_A(3,3)=1$ $\Delta_B(3,3)=2$	$\Delta_A(3,4)=0$ $\Delta_B(3,4)=4$
$u_A=4$	$\Delta_A(4,1)=2$ $\Delta_B(4,1)=0$	$\Delta_A(4,2)=1$ $\Delta_B(4,2)=8$	$\Delta_A(4,3)=0$ $\Delta_B(4,3)=5$	$\Delta_A(4,4)=1$ $\Delta_B(4,4)=5$

losses if either player deviates from its optimal reaction to the other player's strategy could be defined as being close to, or near, a Nash equilibrium. Thus we define a pair of strategies $\{u_A^{NN},u_B^{NN}\}$ as a Near-Nash strategy pair if:

$$J(u_A^{NN},u_B^{NN})=\min_{\{u_A,u_B\}\in U_A\times U_B}\left\{\begin{bmatrix}J_A(u_A^*(u_B),u_B)\\-J_A(u_A,u_B)\end{bmatrix}^2+\begin{bmatrix}J_B(u_A,u_B^*(u_A))\\-J_B(u_A,u_B)\end{bmatrix}^2\right\} \quad (5)$$

We note that this definition includes also, and can be used to compute, the Nash equilibrium. Clearly, when the above minimum is equal to zero, the Near-Nash strategies will coincide with the Nash strategies. To illustrate this, Table 4 shows the values of $J(u_A,u_B)$ for all pair of strategies in Table 1. The Near Nash strategies in this case are $\{u_A^{NN},u_B^{NN}\}$={2,2} resulting in $J(2,2)=0$ which are the same as the Nash strategies for this game.

Table 4: Values of $J(u_A,u_B)$ for all strategies in Table 1

	$u_B=1$	$u_B=2$	$u_B=3$	$u_B=4$
$u_A=1$	$J(1,1)=25$	$J(1,2)=272$	$J(1,3)=365$	$J(1,4)=64$
$u_A=2$	$J(2,1)=17$	$J(2,2)=0$	$J(2,3)=160$	$J(2,4)=40$
$u_A=3$	$J(3,1)=2$	$J(3,2)=64$	$J(3,3)=5$	$J(3,4)=16$
$u_A=4$	$J(4,1)=4$	$J(4,2)=65$	$J(4,3)=25$	$J(4,4)=26$

Now, for the modified game in which the value of $J_B(2,4)$ in Table 1 is altered from 2 to 10 and for which there is no Nash equilibrium, the values of $J(u_A,u_B)$ are tabulated in Table 5.

Table 5: Values of $J(u_A, u_B)$ for all strategies in Table 1 when $J_B(2,4)$ is altered from 2 to 10

	$u_B = 1$	$u_B = 2$	$u_B = 3$	$u_B = 4$
$u_A = 1$	$J(1,1) = 25$	$J(1,2) = 272$	$J(1,3) = 365$	$J(1,4) = 64$
$u_A = 2$	$J(2,1) = 25$	$J(2,2) = 4$	$J(2,3) = 180$	$J(2,4) = 4$
$u_A = 3$	$J(3,1) = 2$	$J(3,2) = 64$	$J(3,3) = 5$	$J(3,4) = 16$
$u_A = 4$	$J(4,1) = 4$	$J(4,2) = 65$	$J(4,3) = 25$	$J(4,4) = 26$

Clearly this table indicates that this game has no Nash equilibrium and the Near Nash strategies in this case are $\{u_A^{NN}, u_B^{NN}\} = \{3,1\}$ corresponding to the smallest value of $J(u_A, u_B)$=2. By using these strategies, each player is guaranteed a loss of no more than 1 if the other player deviates from its optimal reaction. This appears to be a very appropriate strategy when a Nash equilibrium does not exist.

3. INCORPERATING THE NEAR-NASH IN AN AUTONOMOUS BATTLE PLANNER

Modern military conflict combines a near infinite number of strategies and command decisions, as well as considerable heterogeneity and interdependency in unit attributes and pervasive uncertainty in regards to units on both sides. Consequently, it is entirely impractical to implement an autonomous battlefield planner that searches for a Nash equilibrium in the classical sense when it is not known apriori that a Nash equilibrium exists. An alternative approach would be to compute the quantity $J(u_A, u_B)$ for all pairs of strategies and search for either a Nash ($J(u_A, u_B)$=0) or a Near-Nash solution (minimum of $J(u_A, u_B)$).

To illustrate the effectiveness of the Near-Nash equilibrium and to show that it has properties similar to that of the Nash equilibrium we use a simplistic model of battlefield dynamics that corresponds to the Multi-Team Dynamic Weapon Target Allocation Problem (Galati and Simaan 2007). In this model, two or more teams of heterogeneous fighting units must collaborate as a team to destroy enemy units while preserving friendly units over a number of targeting rounds (Figure 1). The decision space for this problem is limited to determining a target for each asset. This is an extension of the Weapon Target Assignment (WTA) problem (Matlin 1970) and the Dynamic Weapon Target Assignment (DWTA) problem (Murphy 1999) in that each unit acts like both a weapon against a unit in the other team and a target for a unit in the other team.

To present a mathematical formulation for this structure, let the two teams be labeled as Blue (B) and Red (R) and let K denote the total number of time steps representing the duration of the battle. Let the number of non-homogeneous fighting units at step k, where k=0, 1,…,K in each team be $N_B(k)$ and $N_R(k)$ respectively.

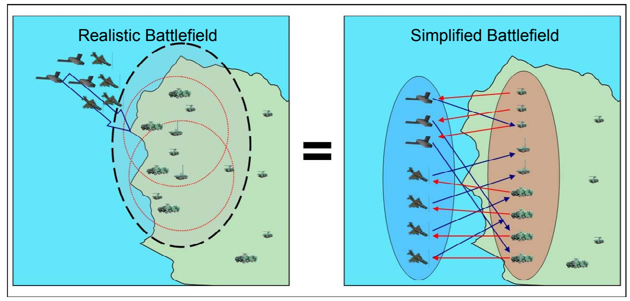

Fig. 1. MT-DWTA simplification of the battlefield

Since every unit in each team is to be assigned a unit on the other team as a target, the number of possible target assignments is $N_B(k)$ for each unit on the Blue team and $N_R(k)$ for each unit on the Red team. These strategies must be selected for battle steps k=0,1,…,K-1. If, at each battle step k, a team chooses a strategy based upon an objective function, we assume that this objective function will take the form of a weighted sum, maximizing both the combined worth of the destroyed units in the other team and the combined worth of the remaining units in that team. Let these objective functions at step k be $J_B(u_B(k), u_R(k))$ for the Blue team and $J_R(u_B(k), u_R(k))$ for the Red team, where $u_B(k) = [u_{B1}(k), u_{B2}(k), .., u_{BN_B}(k)]'$ and $u_R(k) = [u_{R1}(k), u_{R2}(k), .., u_{RN_R}(k)]'$ are $N_B(k)$ and $N_R(k)$ dimensional vectors representing the Blue and Red team's respective target assignment strategies at step k. The i^{th} entry $u_{Bi}(k) \in \{0,1,....,N_R(k)\}$ in $u_B(k) \in U_B(k)$ represents the Red target assigned to the i^{th} Blue unit. A similar representation is also employed for the Red team[i].

In the MT-DWTA, each unit may be valued differently by each team. Let b_i^B denote the worth of the i^{th} Blue unit to the Blue team and let b_i^R denote the worth of that unit to the Red team. Likewise, let r_i^B and r_i^R denote the worth of the j^{th} Red unit to the Blue and Red teams respectively. Assume that the probability of kill of the i^{th} Blue unit against the j^{th} Red unit at the

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[i] The choice $u_{Bi}(k) = 0$ or $u_{Rj}(k) = 0$ implies that no target has been assigned to the i^{th} unit in the Blue team or the j^{th} unit in the Red team. The strategy $u(k) = \underset{\sim}{0} = [0, 0, ..., 0]'$ is used to denote that no unit in a given team has been assigned a target.

k^{th} battle step is $p^B_{i,j}(k)$. Similarly, let the probability of kill of the j^{th} Red unit against the i^{th} Blue unit be $p^R_{i,j}(k)$. Finally, let $B_i(k)$ and $R_j(k)$ denote the probabilities that the i^{th} Blue unit and j^{th} Red unit are alive at the start of the k^{th} battle step. Using these notations we can express the probability of survival of the i^{th} Blue unit and j^{th} Red unit as follows:

$$B_i(k) = B_i(k-1)\prod_{j=1}^{N_R(k)}\left[1 - p^R_{j,i}(k)\delta(i-u_{R_j}(k))R_j(k-1)\right] \quad \text{(6a)}$$

$$R_j(k) = R_j(k-1)\prod_{i=1}^{N_B(k)}\left[1 - p^B_{i,j}(k)\delta\left(j-u_{B_i}(k)\right)B_i(k-1)\right] \quad \text{(6b)}$$

Consequently, the objective functions for the Blue and Red teams can be expressed as:

$$\begin{aligned} J_B(u_B,u_R,k) &= \sum_{i=1}^{N_B(k)} b_i^B B_i(k) - \sum_{j=1}^{N_R(k)} r_j^B R_j(k) \\ J_R(u_B,u_R,k) &= -\sum_{i=1}^{N_B(k)} b_i^R B_i(k) + \sum_{j=1}^{N_R(k)} r_j^R R_j(k) \end{aligned} \quad (7)$$

where the term $\delta(p-q)$ is the Kronecker delta defined by $\delta(p-q) = \begin{cases} 0 & \text{if } p \neq q \\ 1 & \text{if } p = q \end{cases}$, and is used to indicate that unit q of the Blue team has been assigned to target unit p in the Red team

Applying the near-Nash equilibrium defined in (4, 5) to the MT-DWTA objective functions as presented in (7), we are left with the following multi stage optimization:

$$\min_{(u_B,u_R)\in(U_B\times U_R)} \left\{ \begin{aligned} &\left[J_B\left(u_B^*(u_R),u_R\right) - J_B\left(u_B,u_R\right)\right]^2 + \\ &\left[J_R\left(u_B,u_R^*(u_B)\right) - J_R\left(u_B,u_R\right)\right]^2 \end{aligned} \right\} \quad (8)$$

where

$$J_B\left(u_B^*(u_R),u_R\right) = \min_{u_B\in U_B}\left\{\sum_{i=1}^{N_B(k)} b_i^B B_i(k) + \sum_{j=1}^{N_R(k)} r_j^B R_j(k)\right\}$$

$$J_R\left(u_B,u_R^*(u_B)\right) = \min_{u_R\in U_R}\left\{\sum_{i=1}^{N_B(k)} b_i^R B_i(k) + \sum_{j=1}^{N_R(k)} r_j^R R_j(k)\right\}$$

We note that even though the objective functions are evaluated for the predicted assets remaining at the end of a single battle-step, the control vectors $\{u_B,u_R\}$ may extend over multiple battle-steps. For simplicity, in this paper we will only consider two battle-steps, k and k+1, optimizing for the $k+1^{th}$ step (in other words one step look-ahead). Thus each control vector may be expressed:

$$\begin{aligned} u_B \in U_B &\Rightarrow [u_B(k),u_B(k+1)] \in [U_B(k),U_B(k+1)] \\ u_R \in U_R &\Rightarrow [u_R(k),u_R(k+1)] \in [U_R(k),U_R(k+1)] \end{aligned} \quad (9)$$

It has been shown that each of the sub-optimizations in (8) is NP Hard (Murphy 1999). Consequently, we cannot solve (8) exactly[ii]. Previously, we have shown that ULTRA, a neighborhood search technique which attempts to improve a given strategy by modifying the target assignments of one or more assets (Galati et. al. 2003) is capable of quickly determining target assignments that are on average 95% optimal for scenarios approaching 200 individual assets.

To find a Near-Nash strategy for the MT-DWTA, we will use a tit-for-tat or action/re-action search. We first assume a two step strategy by the Red Team[iii], or $\{u_R(k),u_R(k+1)\}^0$. The Blue team then calculates $\{u_B(k),u_B(k+1)\}^0$ using the ULTRA algorithm (Galati et. al. 2003) to find the Optimal Response to Red's initial strategy. Red then calculates $\{u_R(k),u_R(k+1)\}^1$ as the optimal response to $\{u_B(k),u_B(k+1)\}^0$. This process iterates until on of three possible terminating conditions are reached:

$$\begin{aligned} &\{u_B(k),u_B(k+1)\}^\tau = \{u_B(k),u_B(k+1)\}^{\tau-1} \text{ or} \\ &\{u_R(k),u_R(k+1)\}^\tau = \{u_R(k),u_R(k+1)\}^{\tau-1} \\ &\{u_B(k),u_B(k+1)\}^\tau = \{u_B(k),u_B(k+1)\}^{\tau-\theta} \text{ or} \\ &\{u_R(k),u_R(k+1)\}^\tau = \{u_R(k),u_R(k+1)\}^{\tau-\theta} \end{aligned} \quad (10)$$

where $\theta > 2$ and $\tau >$ maximum number of iterations.

Thus the algorithm will terminate in a cycle of either 1, θ, or τ iterations. After one of these three terminating conditions has been reached, we select the pair of strategies within this cycle which are closest to a Nash Equilibrium. Having used this algorithm extensively, we can make several observations that are not analytically provable, but are useful nonetheless:

While (10) is theoretically possible, we find that this algorithm typically converges with $\tau \leq 10$. This is because there are a small number of individual optimal responses for the entire set of adversarial strategies.

We have found that $\theta \leq 3$, though is typically confined to 1 or 2. We note that if the terminating condition in (10) is reached, then the given strategies are by definition a Nash strategy pair as each strategy is an optimal response to the other.

[ii] We note that the inability to find a closed form solution to (8) does not prevent it from acting as a test case for Near-Nash planners. On the contrary, this is representative of the conditions most battlefield planners will have to contend with. By illustrating that near-Nash strategies perform almost, but not quite, as well as pure Nash Equilibriums in a manner similar to the way sub-optimal approximations are almost, but not quite optimal, we further enhance our claim that near-Nash based strategies have many of the properties of Nash equilibrium without the rigorous requirements.

[iii] Though either is acceptable, for simplicity and without loss of generality we will assume that the Red Team provides the initial strategy.

4. EVALUATING THE EFFECTIVENESS OF NEAR NASH STRATEGIES

To verify the effectiveness of the Near Nash equilibrium as a solution concept we will examine interactions of the following three strategy types: Near-Nash against Near-Nash, Near-Nash against Optimal Response (denoted by a * superscript), and Near-Nash against Team Optimal Response (denoted by an *o* superscript). We note that the target assignment vectors are to be selected from discrete spaces labeled $U_B^x(k)$ and $U_R^x(k)$, each containing $S_B^x(k)$ and $S_R^x(k)$ possible target assignments strategies available to each team respectively at step *k*.

A strategy $\{u_B^*(k), u_B^*(k+1)\} \in U_B^*(k)$ is defined as an **Optimal Response** strategy for the Blue team given an announced strategy by the Red team $u_R^A(k)$, over a look ahead horizon *d=1* if at step *k* it satisfies the inequality:

$$\begin{aligned} &J_B\left(\left\{u_B^*(k), u_B^*(k+1)\right\}, \left\{u_R^A(k), u_R(k+1)\right\}\right) \geq \\ &J_B\left(\left\{u_B(k), u_B(k+1)\right\}, \left\{u_R^A(k), u_R(k+1)\right\}\right) \\ &\qquad \forall \left\{u_B^*(k), u_B^*(k+1)\right\} \in U_B^*(k) \end{aligned} \tag{11a}$$

Likewise a strategy $\{u_R^*(k), u_R^*(k+1)\} \in U_R^*(k)$ is defined as an **Optimal Response** strategy for the Red team given an announced strategy by the Blue team $u_B^A(k)$, over a look ahead horizon *d=1* if at step *k* if it satisfies the inequality:

$$\begin{aligned} &J_R\left(\left\{u_B^A(k), u_B(k+1)\right\}, \left\{u_R^*(k), u_R^*(k+1)\right\}\right) \geq \\ &J_R\left(\left\{u_B^A(k), u_B(k+1)\right\}, \left\{u_R(k), u_R(k+1)\right\}\right) \\ &\qquad \forall \left\{u_R^*(k), u_R^*(k+1)\right\} \in U_R^*(k) \end{aligned} \tag{11b}$$

We also note that the Optimal Response strategy is calculated with full knowledge of the adversaries intended strategy. We also note that the Optimal Response strategy for a team depends only on the strategy announced by its opponent for the current battle step. Its objective function can be decoupled during the second battle step.

A strategy $u_B^o(k) \in U_B^0(k)$ is called a **Team Optimal strategy** for the Blue team at step *k* if it is selected such that $J_B(u_B^o(k), \underset{\sim}{0}, k) \geq J_A(u_B(k), \underset{\sim}{0}, k)$ for all $u_B(k) \in U_B^o(k)$. Similarly, a strategy $u_R^o \in U_R^o(k)$ is called a **Team Optimal strategy** for the Red team if it is selected such that $J_R(\underset{\sim}{0}, u_R^o(k), k) \geq J_A(\underset{\sim}{0}, u_R(k), k)$ for all $u_R(k) \in U_R^o(k)$.

We note that the Team Optimal strategy is one that completely ignores the adversarial nature of the other team and considers it only as a set of target units. It represents the standard non-game based solution to the target assignment problem (Murphy 1999).

To demonstrate the effectiveness of near Nash strategies on a MT-DWTA problem, we conducted a series of Monte Carlo simulations on a scenario where a team of 10 Red units were engaged with a team of 10 Blue units. We assumed that the two 10 10 matrices of probabilities of kill of Blue against Red and Red against Blue have entries that are random numbers uniformly distributed in the interval [0, 1]. The objective functions for each team were structured as described in equations (7) and the unit worth values $b_i^B, b_i^R, r_j^B, r_j^R$ were also randomly and independently selected in the range [0, 1] with uniform probability distributions[iv]. To obtain valid aggregate results, we performed 30,000 runs for each of the combination of strategies and averaged the results. These runs differed in that all random numbers were selected for each run using a different seed. The results of this simulation are tabulated in 6.

Table 6 – Comparing Near-Nash vs. other strategies

		Combination of Strategies Employed		
		near-Nash (Blue) vs near-Nash (Red)	near-Nash (Blue) vs Optimal Response (Red)	near-Nash (Blue) vs Team Optimal (Red)
% of initial forces remaining	Blue	9%	8%	12%
	Red	9%	10%	7%

Examining Table 6, we can see that the case of Near-Nash against Near-Nash yields results very similar to the results of Near-Nash versus Optimal Response. In contrast, we see that there is a more substantial difference between the results of Near-Nash versus Near-Nash and those obtained from Near-Nash versus Team Optimal strategies. In a true Nash equilibrium, because no team has an incentive to alter their strategy, the Optimal Response strategy will yield identical results to that of the Nash Equilibrium. Thus we can conclude from the simulations that the Near-Nash approach yields results very close to those expected from a true Nash Equilibrium even though such an equilibrium does not always exist or cannot be found.

5. CONCLUSION

Combat systems of the future will begin to deploy unmanned assets in ever larger numbers as the capabilities of unmanned systems evolve. This migration from manned to unmanned will result in many changes in the militaries force structure. One major change will be the relationship between commanders and unmanned assets. Whereas today's military often assigns multiple commanders to a single unmanned asset, the force

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[iv] We acknowledge that military assets do not have random probability of kill vectors. Various assets tend to have distinct strengths and vulnerabilities. However, we find that Nash strategies fare much better in structured scenarios (Galati and Simaan 2007). Therefore, using random probability of kill matrices represents a worst case scenario for near-Nash Strategies.

structure of the future will require a single commander to intelligently control multiple autonomous assets.

One commander is not capable of performing all of the functions necessary to control multiple autonomous assets in the same manner as today's unmanned systems. Most likely, the commander of the future will rely on battle planning software to augment their abilities and to automate many planning functions. However, there is a large technology gap between what is available now and what is needed in the future. A great deal of work must be done to bridge this gap.

Planning aids based upon game theoretic concepts, the Nash equilibrium in particular, are one promising avenue of research. However, because of the uncertainty as to the existence of the equilibrium point, and the increased domain knowledge required to conduct such an analysis, there is often a desire to use simple, naïve strategy options that do not reason about possible adversarial actions.

In this paper we sought to answer these common criticisms and to justify future research into game theoretic planning for unmanned assets. We introduced the concept of the Near-Nash strategies to overcome the possibility that a unique Nash may not exist. We applied the Near Nash concept to the MT-DWTA, a representative example of a battle space, using the ULTRA algorithm and a tit-for-tat action/reaction type iterative search. We then compared the performance of a Near-Nash based strategy to the Optimal Response, Team optimal, and Near-Nash strategies. Using a series of Monte Carlo simulations, we demonstrated that the Near-Nash strategies are justifiable in that they yield results comparable to what a genuine Nash equilibrium would yield.

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